

Glass phases of flux lattices in layered superconductors

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We study a flux lattice which is parallel to superconducting layers, allowing for dislocations and for disorder of both short wavelength and long wavelength. We find that the long wavelength disorder of strength $\tilde{\Delta}$ has a significant effect on the phase diagram – it produces a first order transition within the Bragg glass phase and leads to melting at strong $\tilde{\Delta}$. This then allows a Friedel scenario of 2D superconductivity.

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The influence of disorder on the behavior of flux arrays in superconductors is intensively studied in recent years. A model suggested by Larkin¹ in which random forces act independently on each vortex was applied to conventional superconductors. Using the arguments of Ref.¹ it was shown that the flux array is collectively pinned², forming a vortex glass (VG) phase^{4,5,6}. However, experiments^{7,8} on weakly disordered samples, reveal long-range order of the flux array, much beyond the characteristic Larkin length. This phenomena is accounted for^{9,10,11} by an elastic dislocation-free theory of the flux lattice in weak random potential which predicts an algebraic decay of the translational order and the existence of divergent Bragg peaks in the glass phase (Bragg glass). It was argued^{10,11} that the Bragg glass phase is stable against formation of dislocations in a finite range of the phase diagram.

The phenomena of melting of the flux lattice is of considerable interest. Melting has been observed^{7,8,12,13,14,15} as either a transition into a flux liquid phase or into a glass phase with a higher critical current. A model for melting, allowing for both disorder and dislocations^{11,16} was recently studied. The model considers flux lines parallel to and confined between superconducting layers and allows for dislocations. This model was studied without disorder¹⁷ leading to flux melting at a critical temperature T_c which is about factor ~ 2 from the solution of the more fundamental system in terms of superconducting phases^{18,19}; the latter allows also for flux loops and overhangs. Since disorder has drastic effects on melting we expect that the model in terms of flux displacement is a reasonable starting point.

The solution of Carpentier, Le-Doussal and Giamarchi (CLG)¹⁶ has shown explicitly that short wavelength disorder combined with dislocations leads to melting at a finite value of the disorder strength. CLG have used Replica Symmetry Breaking (RSB) methods as well as Renormalization Group (RG). They have also shown^{16,20} that this melting is compatible with a Lindemann criterion.

In the present work we allow for an additional term in the CGL model. This term is generated by RG within the CLG model and leads to significant effects on the phase diagram. Using RSB methods, we show that long wavelength disorder of strength $\tilde{\Delta}$ leads to a first order transition within the Bragg glass phase and as $\tilde{\Delta}$ increases it leads to melting. We find that the $\tilde{\Delta}$ induced melting is inconsistent with a universal Lindemann criterion. Finally we consider the quest for the Friedel scenario²¹ in which a layered superconductor becomes a set of decoupled two-dimensional (2D) superconductors. This scenario fails in pure superconductors^{22,23}, but is possible with some constraints in parallel fields¹⁹ and in special models²⁴. With disorder which affects interlayer coupling the Friedel scenario becomes feasible in presence of a melted flux array.

The model^{11,16,17} consists of layers with interlayer spacing l where modulation in the flux line density couples to a random potential. We consider a Hamiltonian with two types of random potentials,

$$H = \int d^2r \sum_i \left[\frac{c}{2} (\nabla \Phi_i(\mathbf{r}))^2 - \eta_i(\mathbf{r}) \nabla \Phi_i(\mathbf{r}) - \mu \cos(\Phi_i(\mathbf{r}) - \Phi_{i+1}(\mathbf{r})) \right. \\ \left. - 2Re(\zeta_i(\mathbf{r}) e^{i\Phi_i(\mathbf{r})}) \right] \quad (1)$$

with Gaussian disorder correlations $\langle \zeta_i(\mathbf{r}) \zeta_j(\mathbf{r}') \rangle = 4Tg\delta_{i,j}\delta(\mathbf{r} - \mathbf{r}')$ and $\langle \eta_i(\mathbf{r}) \eta_j(\mathbf{r}') \rangle = T\Delta\delta_{i,j}\delta(\mathbf{r} - \mathbf{r}')$ where T is the temperature. Here $\Phi_i(\mathbf{r})$ stands for in-plane displacement of the vortex line in the i -th layer, c is an in-plane elastic constant, g measures disorder with Fourier component $\approx 2\pi/a$ where a is the flux periodicity parallel to the layers, while Δ measures long wavelength disorder¹⁰. The μ term is the coupling between layers which allows for dislocations. In the pure system¹⁷ thermal fluctuations lead to melting, i.e. μ is renormalized to zero, at $T_c = 4\pi c$.

Dimensional Imry-Ma arguments are useful to check the stability of an ordered phase which is a d dimensional elastic medium. In a domain of size L the elastic energy is $\propto L^{d-2}$, the short wavelength disorder (after averaging the square) is $\propto L^{d/2}$ while the long wavelength disorder is $\propto L^{(d-2)/2}$. Thus short wavelength disorder is relevant at $d < 4$, while the long wavelength disorder is marginal only at $d = 2$, i.e. the latter is consistent with long range order in $d=3$.

To average over disorder we start with the replicated version of Hamiltonian (Eq.(1)) which includes all relevant terms generated by renormalization,

$$\begin{aligned}
H = & \int d^2r \left\{ \frac{c}{2} \sum_{i,a} [(\nabla \Phi_i^a(\mathbf{r}))^2 - \mu \cos(\Phi_i^a(\mathbf{r}) - \Phi_{i+1}^a(\mathbf{r}))] \right. \\
& \sum_{i,a,b} \left[\frac{\Delta}{2} \nabla \Phi_i^a(\mathbf{r}) \nabla \Phi_i^b(\mathbf{r}) - \gamma \cos(\Phi_i^a(\mathbf{r}) - \Phi_{i+1}^a(\mathbf{r}) - \Phi_i^b(\mathbf{r}) + \Phi_{i+1}^b(\mathbf{r})) \right. \\
& \left. \left. - g \cos(\Phi_i^a(\mathbf{r}) - \Phi_i^b(\mathbf{r})) \right] \right\}
\end{aligned} \tag{2}$$

where $a = 1 \dots n$ is the replica index. Note in particular the γ term which was not considered by GLC; this term is generated in second order RG from the μ term. Since it couples different replicas it can lead to RSB, i.e. this term leads to distinct phenomena and should be included in the full Hamiltonian.

We consider the variational free energy $F_{var} = F_0 + \langle H - H_0 \rangle$ with

$$H_0 = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \int_{-\pi}^{\pi} \frac{dq_z}{2\pi} G_{ab}^{-1}(\mathbf{q}, q_z) \Phi_i^a(\mathbf{q}, q_z) \Phi_i^b(-\mathbf{q}, -q_z) \tag{3}$$

where the Greens' function $G_{ab}^{-1}(\mathbf{q}, q_z)$ is determined by an extremum condition of F_{var} and \mathbf{q}, q_z are Fourier variables for \mathbf{r} and i , respectively.

Defining the inverse Green's function in the form $G_{ab}^{-1}(\mathbf{q}, q_z) = \delta_{ab} G_0^{-1}(\mathbf{q}, q_z) - \sigma_{ab} - \Delta q^2$ with $\sum_a \sigma_{ab} = 0$ and $\sigma_{ab} = 2(1 - \cos q_z) \sigma_{ab}^\gamma + \sigma_{ab}^g$ we obtain the the self-consistent equations in the form

$$G_0^{-1}(q, q_z) = cq^2 + 2\tilde{\mu}(1 - \cos q_z) \tag{4}$$

$$\sigma_{ab}^g = 2g \exp[-\frac{1}{2}B_{ab}^g] \tag{5}$$

$$\sigma_{ab}^\gamma = \gamma \exp[-B_{ab}^\gamma] \tag{6}$$

$$\tilde{\mu} = \mu \exp[-\frac{1}{2}B_{aa}] \tag{7}$$

where we define

$$B_{ab}^g = 2T \sum_{\mathbf{q}, q_z} [G_{aa}(\mathbf{q}, q_z) - G_{ab}(\mathbf{q}, q_z)] \tag{8}$$

$$B_{ab}^\gamma = 2T \sum_{\mathbf{q}, q_z} (1 - \cos q_z) [G_{aa}(\mathbf{q}, q_z) - G_{ab}(\mathbf{q}, q_z)] \tag{9}$$

$$B_{aa} = 2T \sum_{\mathbf{q}, q_z} (1 - \cos q_z) G_{aa}(\mathbf{q}, q_z). \tag{10}$$

Here $\tilde{\mu}$ is the renormalized coupling between layers which is determined by the diagonal B_{aa} ; $\tilde{\mu} = 0$ signals a 2D phase, i.e. correlations in the z direction are lost and the flux lattice has melted.

We study the full RSB solution of the saddle point equations (5-7). The method of RSB²⁵ employs a representation of hierarchical matrices such as $\sigma_{ab}^{g,\gamma}$ in term of functions $\sigma_{g,\gamma}(u)$, and similarly $B_{ab}^{g,\gamma}$ is represented by $B_{g,\gamma}(u)$ with $0 < u < 1$.

We define two order parameters for RSB, $[\sigma_{g,\gamma}](u) = u\sigma_{g,\gamma}(u) - \int_0^u \sigma_{g,\gamma}(v)dv$. Using the inversion formula²⁵ for G_{ab} and integrating over momenta we obtain

$$\frac{1}{2}B_{g,\gamma}(u) = \frac{1}{u}g_{g,\gamma}(u) - \int_u^1 \frac{ds}{s^2}g_{g,\gamma}(s) \tag{11}$$

$$\begin{aligned}
g_g(u) &= -\tilde{T} \ln \left[\frac{m}{\Lambda} (\rho + 1 + w) \right] \\
g_\gamma(u) &= g_g(u) - \tilde{T}(\rho + 1 + w)^{-1}
\end{aligned} \tag{12}$$

Here $\rho(u) = [w(u)(w(u) + 2)]^{1/2}$, $w(u) = ([\sigma_g](u))/2m(u)$, $m(u) = \tilde{\mu} + [\sigma_\gamma](u)$, Λ is a cutoff of cq^2 ($\Lambda \gg m(u)$, $[\sigma_g](u)$) and $\tilde{T} = T/T_c$ with $T_c = 4\pi c$.

By differentiating Eqs.(5-6) we obtain two coupled differential equations for the RSB functions $m(u)$ and $w(u)$

$$\begin{aligned}
\frac{2\tilde{T}}{u} \frac{dm}{du} &= \frac{d}{du} \left[\frac{m\rho(1 + w + \rho)}{\rho(1 + w + \rho) + Q(w + \rho)} \right] \\
\frac{\tilde{T}}{u} \frac{dm}{du} (Q + w) &= \frac{d}{du} \left[\frac{m\rho(Q + w)}{Q + \rho} \right]
\end{aligned} \tag{13}$$

where $Q(u) = m(dw/du)/(dm/du)$.

A general solution of these equations is rather difficult, so at first we consider special limits. When $\gamma = 0$ we recover the CLG solution¹⁶. Within the 3D Bragg glass phase $\langle [\Phi_i(r) - \Phi_i(0)]^2 \rangle \sim \ln r$ so that positional correlations decay algebraically and long range order is weakly destroyed. The Bragg glass phase undergoes a continuous melting transition (for $\Delta = 0$) at $g/\mu = 2/e\tilde{T}$ as shown in the $\tilde{\Delta} = 0$ plane of Fig. 1; for $\Delta \neq 0$ the transition becomes first order. Thus, the Bragg glass phase, due to both disorder and dislocations, melts into a 2D phase with $\tilde{\mu} = 0$.

Consider next the case $g = 0$, hence $w(u) = 0$; the solution in this case is formally similar to that of a 2D disordered Josephson junction²⁶. Eq. (13) yields then $(1 - 2\tilde{T}/u)m'(u) = 0$, i.e. $m(u)$ is a one step function, with the step at $u = 2\tilde{T}$. Since $u < 1$ the onset of this solution is at $\tilde{T} = 1/2$. i.e. at $T = T_c/2$. Eqs. (6,7) determine the jump in $[\sigma_\gamma](u)$ from zero ($u < 2\tilde{T}$) to a value σ_γ^0 at $2\tilde{T} < u < 1$, where

$$\frac{\tilde{\mu} + \sigma_\gamma^0}{\Lambda} = e^{-1} \left(4e\tilde{T} \frac{\gamma}{\Lambda} \right)^{1/(1-2\tilde{T})} \quad (14)$$

$$\frac{\tilde{\mu}}{\Lambda} = e^{-1} \left[e^{\tilde{\Delta}+1/2} \left(4e\tilde{T} \frac{\gamma}{\Lambda} \right)^{-1/2} \frac{\mu}{\Lambda} \right]^{1/(1/2-\tilde{\Delta})} \quad (15)$$

where $\tilde{\Delta} = 4\pi\tilde{T}^2\Delta$. This solution is valid for $\tilde{\Delta} < 1/2$ and $\tilde{T} < 1/2$. For γ of order μ , near the $\tilde{T} = 1/2$ transition $\tilde{\mu}$ is finite while $\tilde{\mu} + \sigma_\gamma^0$ vanishes. Thus $\sigma_\gamma^0 < 0$ is finite up to $\tilde{T} = 1/2$ and vanishes at $\tilde{T} > 1/2$, i.e. the transition is of first order. When $\tilde{\Delta} > \tilde{T}$ within this phase σ_γ^0 changes sign and becomes positive.

The phase at $\tilde{\Delta} < 1/2$ and $\tilde{T} < 1/2$ is a coexistence phase—it has both long range order ($\tilde{\mu} \neq 0$) and glass order ($\sigma_\gamma^0 \neq 0$). (As noted above this is consistent with the Imry-Ma argument). At $\tilde{\Delta} = 1/2$ we find a disorder driven transition where $\tilde{\mu}$ vanishes continuously, leading to a 2D glass phase at $\tilde{\Delta} > 1/2$.

We note also that a replica symmetric solution is possible for $1/2 < \tilde{T} < 1$ with

$$\frac{\tilde{\mu}}{\Lambda} = e^{-1} \left(e^{\tilde{\Delta}+1/2} \frac{\mu}{\Lambda} \right)^{1/(1-\tilde{T}-\tilde{\Delta})} \quad (16)$$

i.e. $\tilde{\mu} \neq 0$ for $\tilde{T} + \tilde{\Delta} < 1$, $\tilde{T} > 1/2$ as shown in Fig. 1. Comparison with Eqs. (14-15) shows that $\tilde{\mu}$ is also discontinuous at the $\tilde{T} = 1/2$ transition.

Finally we consider the case where both g and Δ are finite. We can demonstrate the existence of a first order transition at small g by showing a coexistence of two solutions. The first solution is an expansion near the $g = 0$ solution, i.e. $w(u) \ll 1$ with $[\sigma_g] = O(g)$, $[\sigma_\gamma] = O(1)$; for $u < \tilde{T}/2$ the solution for $[\sigma_g]$ is similar to the $\gamma = 0$ case, i.e. $[\sigma_g](u) \sim u^2$ for small u , consistent with Bragg glass correlations. This solution is valid (assuming $\mu \sim \gamma$) if

$$g/\Lambda \ll (\gamma/\Lambda)^{\frac{1-3\tilde{\Delta}+2\tilde{T}\tilde{\Delta}}{(1-2\tilde{T})(1-2\tilde{\Delta})}} \quad (17)$$

i.e. for weak coupling $g/\Lambda, \gamma/\Lambda \ll 1$ this expansion breaks down close to the transitions at $\tilde{T} = 1/2$ and $\tilde{\Delta} = 1/2$. The second solution is an expansion around the $\gamma = 0$ solution with $[\sigma_\gamma](u) \ll \tilde{\mu}$. This leads to $[\sigma_g] = O(g^2)$, $[\sigma_\gamma] = O(g)$ and is valid for $\tilde{\Delta} < \tilde{T}$. Thus for small g there is a two solution regime which implies a first order transition at some $\tilde{T} \lesssim 1/2$. We indicate this transition by a spaced dashed line in Fig. 1, though we do not know its precise location.

As shown in Fig. 1, we find that the main feature of the CLG scenario is valid—for small disorder the Bragg glass is stable, while at large disorder, which can have either short or long wavelength, dislocations are enhanced by disorder and lead to melting.

These analytic results for melting allow us to test the Lindemann criterion, which is of common use³. For the $\gamma = 0$ case, CLG consider a Lindemann criterion of the form^{16,20} $\langle [\Phi_{i+1}(\mathbf{r}) - \Phi_i(\mathbf{r})]^2 \rangle = c_L^2$, with average done in the elastic limit, i.e. the cosine of the μ term in Eq. (1) is expanded. This criterion leads¹⁶ to a reasonable value of $c_L \lesssim 1$. For the $g = 0$ case an elastic limit leads to an expansion of both the μ and γ terms in Eq. (2) so that RSB is not induced. Since long range order is present the Lindemann criterion is $\langle \Phi_i^2(r) \rangle = c_L^2$; however the replica symmetric solution yields $\langle \Phi_i^2(r) \rangle = (\tilde{\Delta} + \tilde{T}) \ln(\Lambda/\mu)$, i.e. at melting $c_L^2 \approx \ln(\Lambda/\mu)$. since Λ/μ depends on the anisotropy of the system the Lindemann number c_L is non-universal.

In order to relate the phase diagram to the actual magnetic field B we need to identify c by the elastic constants^{3,27} which are dispersive, $c_{44}^{\parallel} \approx c_{11}^{\parallel} = (B^2/4\pi)/(1 + \lambda_c^2 q^2 + \lambda^2 q_z^2)$; here λ, λ_c are penetration lengths and $\epsilon = \lambda/\lambda_c < 1$ is the anisotropy (c_{44}^{\parallel} has a smaller second term which is neglected here). The behavior near melting is dominated by $q \rightarrow 0$ and $q_z \approx 1/l$. The lattice periodicities satisfy $l = a\epsilon$ if $l > d$ for weak fields, i.e. $B = \phi_0/a^2\epsilon < \phi_o\epsilon/d^2$ (d is the spacing of the superconducting layers, which is the lower bound on the interlayer spacing l of the flux lattice, and ϕ_0 is the flux quantum), or $l = d$ for strong fields, $B = \phi_0/ad > \phi_o\epsilon/d^2$. By rescaling the x, z coordinates we identify

$$\begin{aligned}
4\pi c &\approx a\phi_0^2/(4\pi^2\lambda\lambda_c) \sim B^{-1/2} & B < \phi_0\epsilon/d^2 \\
4\pi c &\approx d\phi_0^2/(4\pi^2\lambda^2) & B > \phi_0\epsilon/d^2.
\end{aligned} \tag{18}$$

T_c ($= 4\pi c$) is smallest for large fields, but even then its value is $< 100K$ for typical CuO_2 superconductors only very near the superconducting transition where λ diverges. Thus it is the disorder induced melting which is relevant to experimental data.

Since Δ couples to $\nabla\Phi_i(r)$ ¹⁰ it is B independent so that $\tilde{\Delta} = \Delta/4\pi c^2 \sim B$ for small fields and $\tilde{\Delta} \sim \text{constant}$ for strong fields. Thus the Δ induced melting can be induced by increasing the magnetic field if $\tilde{\Delta} = 1/2$ is achieved for weak fields. On the other hand, for the g induced melting $g \sim 1/a^{210}$, and by using the c_{66} elastic constant²⁸ we identify $\mu = a^2\phi_0 B\epsilon^3/[4\pi^2l(8\pi\lambda)^2]$. For weak fields the melting temperature is $T_m^{-1} = eg/(8\pi c\mu) \sim B^{3/2}$ while for strong fields $T_m^{-1} \sim B^3$. In this case melting is induced by increasing B for both weak or strong fields.

We note finally that any disorder which melts a flux lattice parallel to superconducting layers is a route to the Friedel scenario²¹. In this scenario the layers remain superconducting so that the system behaves as a 2D superconductor. This scenario is not valid in pure layered superconductor^{22,23}, it is valid for parallel fields with restricted system parameters (e.g. large vortex core energy)¹⁹ or in special models²⁴. In the present study, disorder which affects interlayer coupling leads to melting which decouples the layers; assuming weak intralayer disorder the layers remain superconducting. Thus we have a model system which exhibits the Friedel scenario.

In conclusion we have shown that a new interaction term, generated by RG, leads to a significant role of the long wavelength disorder. This interaction extends the CLG results to the more complex phase diagram of Fig.1. We find that the Bragg glass is stable for weak disorder of either short or long wavelength. The long wavelength disorder induces a first order transition within the Bragg glass phase; it also leads to melting which is inconsistent with a Lindemann criterion. We propose that experiments with parallel magnetic fields can test the present theory of melting as well as test the possibility of 2D superconductivity.

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Figure Caption

Phase diagram of layered flux lattices with disorder of short wavelength (with strength g) and of long wavelength (with strength $\tilde{\Delta}$). The dashed line is the line of first order transition (discontinuity of the interlayer coupling and of the glass order); the spaced dashed line is its approximate extension to $g \neq 0$. The 3D glass regime at $g = 0$ has long range order while the 3D Bragg glass at $g \neq 0$ has algebraically decaying positional order.

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